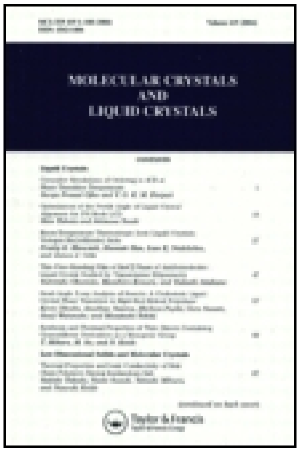

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Second Order Elasticity and Critical Thickness of Hybrid Aligned Nematics Strongly Anchored on the Planar Side

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SECOND ORDER ELASTICITY AND CRITICAL THICKNESS OF HYBRID ALIGNED NEMATICS STRONGLY ANCHORED ON THE PLANAR SIDE.

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Abstract The second order bulk elastic constants, characterizing the squares of second order derivatives of the director, deeply influence the behavior of weakly anchored nematics. Here the critical thickness of a Hybrid Aligned Nematic cell, strongly anchored on the planar side, is calculated as a function of the anchoring strength at the homeotropic side, of the mixed splay-bend, and of the second order bulk elasticity.

INTRODUCTION

The Hybrid Aligned Nematic (HAN) cell, i.e. a cell with homeotropic anchoring at one wall and planar homogeneous anchoring at the other one, was early proposed by Matsumoto et al.[1] as a suitable tool for obtaining coloured displays.

But several authors in the last years showed the fundamental interest of the HAN cell, either to measure the splay-bend elastic ratio K_{11}/K_{33} by means of dielectric[2] and optical methods[3] or to study flexoelectricity[4]. Moreover, great attention has been devoted to non-linear optical reorientation in HAN structures[5], considering also a finite boundary energy[6].

We stress the fact, that the presence of a weak anchoring deeply influences the stability of a HAN cell: for instance, if we consider a weak anchoring on the homeotropic boundary, a HAN cell can be stable only for a thickness d greater than a critical value d_c.

The HAN critical thickness has been approximately evaluated for the first time by Hochbaum and Labes[7], and predicted in a rigorous way by Barbero and Barberi[8], but neglecting the surface-like elasticity due to mixed splay-bend contribution. Moreover, the first experimental evidence of the existence of the critical thickness has been reported in ref.[9], and the behavior of d_c as a function of the elastic ratio

K_{11}/K_{33} has been discussed in ref.[10].

The analysis of ref.[8] was correct from the point of view of Oseen-Frank linear elastic theory: but if we generalize the elastic theory[11], by considering also the effect of the mixed splay-bend and of second order elastic constants, coming from squares of second order derivatives of the director **n** [12], the critical thickness can be shown to depend strictly on the new bulk second order elasticity.

In the present paper such a dependence of the critical thickness is calculated, by restricting ourselves to the hypothesis of one bulk second order elastic constant[13].

THEORY

Let us consider a HAN cell weakly anchored only at the homeotropic wall, $z = 0$, while at the planar homogeneous substrate, $z = d$, the anchoring is strong (see figure 1: z is the co-ordinate normal to the walls).

By assuming that $\theta(z)$ is the tilt angle, which the director **n** forms with the x-axis (parallel to the prefixed orientation at $z = d$), the distortion free energy of the cell in the generalized elastic theory[13] is given by:

$$F = \tfrac{1}{2} K_{11} \int_0^d \left[(1 - k \sin^2 \theta)\, \theta'^2 + \varkappa\, \theta''^2 \right] dz + \\ + \tfrac{1}{2} w_0 \cos^2 \theta_0 - \tfrac{1}{2} K_{13}\, \theta'_0 \sin 2\theta_0 \qquad (1)$$

where $k = 1 - (K_{33}/K_{11})$ is the bulk principal elastic anisotropy, $\varkappa = K^*/K_{11}$ is the elastic ratio between the second order bulk- and the splay- elastic constant, w_0 is the anchoring strength at the homeotropic substrate[14], and K_{13} is the mixed splay-bend elastic constant[15], the prime meaning the derivative with respect to z. By introducing the reduced co-ordinate $\eta = z/d$, the Euler-Lagrange equation writes:

$$b^2 \ddddot{\theta} - (1 - k \sin^2 \theta)\, \ddot{\theta} + (k/2)\, \dot{\theta}^2 \sin 2\theta = 0 \qquad (2)$$

where $b = (K^*/K_{11})^{\frac{1}{2}}/d$, and $\dot{\theta} = d\theta/d\eta$.

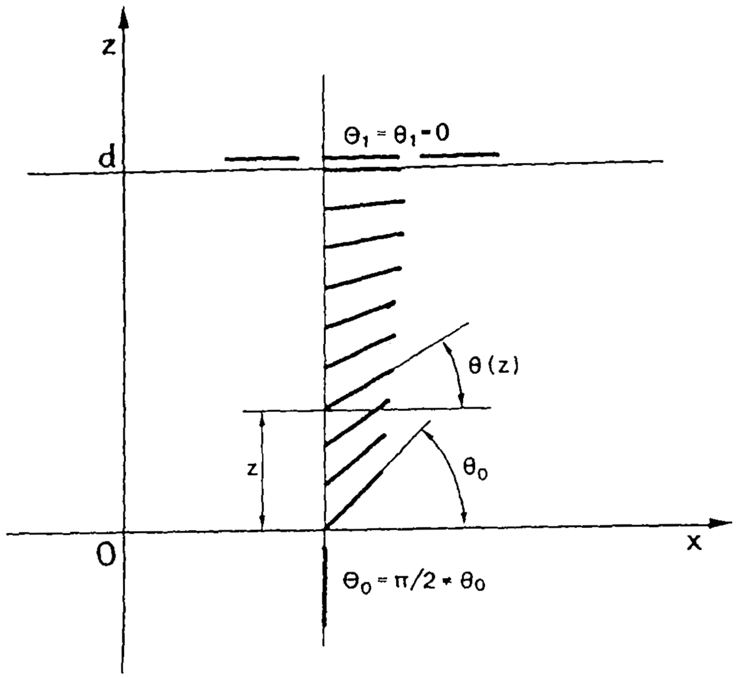

FIGURE 1. Hybrid Aligned Nematic (HAN) cell, weakly anchored at the homeotropic wall ($z = 0$, easy direction $\Theta_0 = \pi/2$), and strongly anchored at the planar one ($z = d$, easy direction $\Theta_1 = 0$).

At the thereshold, for $d \rightarrow d_c$, $\theta \rightarrow 0$: thus eq. (2) becomes

$$b^2 \ddot{\ddot{\theta}} - \ddot{\theta} = 0 \qquad (3)$$

independent of the elastic anisotropy k, as expected.

The solution of eq. (3)

$$\theta(\eta) = C_1\eta + C_2 + \alpha \operatorname{sh}(\eta/b) + \beta \operatorname{ch}(\eta/b) \qquad (4)$$

shall satisfy the linearized boundary conditions:

$$\begin{cases} b^2\,\dddot{\theta}_0 - (1+R)\,\dot{\theta}_0 - \mathscr{L}_0\,\theta_0 = 0 \\ b^2\,\ddot{\theta}_0 + R\,\theta_0 = 0 \\ \theta_1 = 0 \\ \ddot{\theta}_1 = 0 \end{cases} \tag{5}$$

where $R = K_{13}/K_{11}$ is the surface-bulk elastic ratio, $\mathscr{L}_0 = K_{11}/(w_0\,d) = L_0/d$ is the reduced de Gennes - Kleman[16] extrapolation length at the homeotropic wall, and the subscripts o, 1 are relevant to $\eta = 0$, $\eta = 1$, respectively.

By substituting the solution (4) into the eq. (5) we deduce

$$\begin{vmatrix} 1 & 1 & sh(1/b) & ch(1/b) \\ 1+R & \mathscr{L}_0^{-1} & R/b & \mathscr{L}_0^{-1} \\ 0 & 0 & sh(1/b) & ch(1/b) \\ 0 & R & 0 & 1+R \end{vmatrix} = 0 \tag{6}$$

since the coefficient determinant must be zero, to avoid trivial results.

Thus the critical thickness of the HAN-cell is obtained as

$$d_c = (1+R)^2 \,/\, (L_0^{-1} + R^2/\,\delta\,) \tag{7}$$

$\delta = \varkappa^{\frac{1}{2}}$ being a characteristic length, of the order of the molecular interaction, which may reasonably range between ~ 100 Å and ~ 1000 Å.

DISCUSSION

First we note that eq. (7) is not a trivial application of the rule found in ref. [13], which is valid only for cells either homeotropic or homogeneous planar, but anyway with the two substrates having the same prefixed orientations (easy directions). In fact, $d_c = L_0$ being the critical thickness in the linear elastic theory[8], the above rule would give $d_c = K_{11}/w'_0$, with $w'_0 = (w_0 - K_{11} R^2/\delta)/(1 - R)^2$: thus eq. (7) would be $d_c = (1 - R)^2/(L_0^{-1} - R^2/\delta)$, different from the actual one.

Furthermore, d_c strictly depends on L_0, R, and δ : hence optical path measurements performed on a wedge-shaped cell, like that of Barbero and Durand[9], give information on (R, δ), provided the splay constant and the anchoring strength are known, for instance through experiment of polarized light transmission[17].

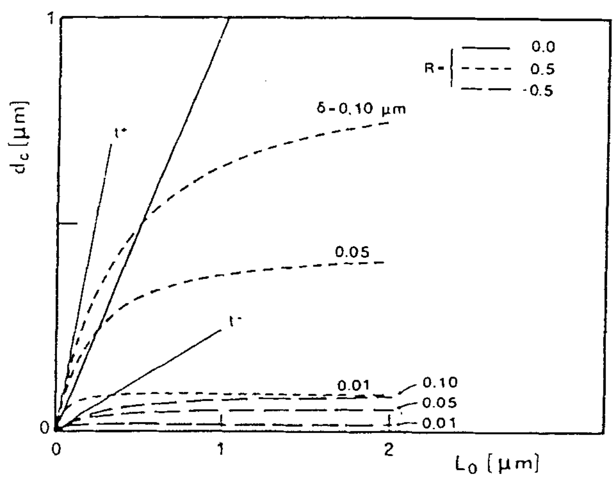

FIGURE 2. HAN- critical thickness d_c vs. the extrapolation length L_0 at the homeotropic wall, for three different values of the characteristic interaction length $\delta = (K^*/K_{11})^{\frac{1}{2}}$ = (0.01, 0.05, 0.10) μm and of the surface-bulk elastic ratio $R = K_{13}/K_{11}$ = -0.5, 0.0, 0.5. Note that the curves slopes at $L_0 = 0$ are independent of δ, being equal to $(1 + R)^2$: $t^{\pm}$ are the tangent in the origin to the curves R = ±0.5, respectively, whereas the curve $d_c(L_0)$ for R = 0.0 is a straight line. Thus it is possible, almost in principle, to measure R independently of δ.

How to determine R and δ separately? Let us observe that the initial slope of the characteristic $d_c(L_o)$ is given by:

$$(\partial d_c / \partial L_o)_{L_o=0} = (1 + R)^2 \quad (8)$$

Hence a set of measurements, performed on wedge-shaped cells with the homeotropic wall having various anchoring strength, is able to determine R , and consequently K_{13} (see figure 2). Of course it is necessary to know K_{11}, and the anchoring strength w_o must be accurately controlled: the latter condition is not a simple task, at the present state-of-the-art.

Afterwards, by substituting the obtained value of R into eq. (7), an indirect measurement of δ can be performed.

Figure 3a) shows the behavior of $d_c(L_o)$ for δ = 500 A and $-1 \leqslant R \leqslant 0$; moreover, in figure 3b) the same function is represented, for the identical value of δ and $0 \leqslant R \leqslant 1$.

It is straigthforward to observe that just one measurement of d_c give no sufficient information, even if δ would be known, since each point in the phase-plane (L_o, d_c) belongs to two curves $d_c(L_o)$, characterized by a negative and a positive value of R, respectively: a complete set of experimental points (L_o, d_c) to be fitted are required.

Finally, it is noticeable that, should the homeotropic wall have very weak anchoring strength (w_o 0), so eq. (7) would become

$$d_c^{free} = (1 + R^{-1})^2 \delta \quad (9)$$

whereas in this case the linear elastic theory would give $d_c^{free} = \infty$

This means that K_{13} destabilizes the homogeneous planar anchoring, in competition with the effect of δ (i. e. of K*): the actual d_c^{free} is enhanced as well as K* increases, for a given value of K_{13}.

Acknowledgments

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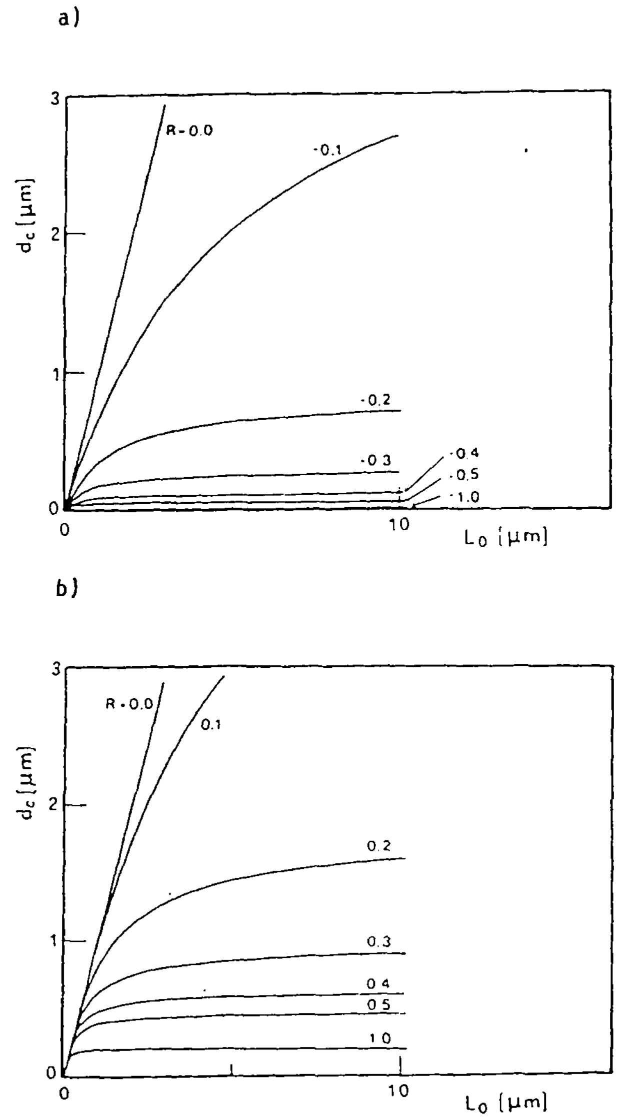

FIGURE 3. HAN-critical thickness d_c as a function of the extrapolation length L_o in the range 0, 10 μm for a given value of the characteristic length δ = 0.05 μm. In **a)** the parameter $R = K_{13}/K_{11}$ ranges from -1 to 0, whereas in **b)** it ranges from 0 to 1. Note that each point of the phase-plane (L_o, d_c) below the curve R = 0.0 belongs to one curve of **a)** and to another curve of **b)**: the ambiguity of finding the actual value of R can be solved only through a set of measurements on cells with various extrapolation lengths.

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